Selected topics in Planck-scale physics

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Abstract

We review a few topics in Planck-scale physics, with emphasis on possible manifestations in relatively low energy. The selected topics include quantum fluctuations of spacetime, their cumulative effects, uncertainties in energy-momentum measurements, and low energy quantum-gravity phenomenology. The focus is on quantum-gravity-induced uncertainties in some observable quantities. We consider four possible ways to probe Planck-scale physics experimentally: 1. looking for energy-dependent spreads in the arrival time of photons of the same energy from GRBs; 2. examining spacetime fluctuation-induced phase incoherence of light from extragalactic sources; 3. detecting spacetime foam with laser-based interferometry techniques; 4. understanding the threshold anomalies in high energy cosmic ray and gamma ray events. Some other experiments are briefly discussed. We show how some physics behind black holes, simple clocks, simple computers, and the holographic principle is related to Planck-scale physics. We also discuss a formulation of the Dirac equation as a difference equation on a discrete Planck-scale spacetime lattice, and a possible interplay between Planck-scale and Hubble-scale physics encoded in the cosmological constant (dark energy).

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Planck was constant.

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I. INTRODUCTION

Planck's constant $\hbar$, Newton's constant G, and the speed of light c can be combined to form the Planck time $t_P = (\hbar G/c^5)^{1/2} \sim 10^{-44}s$, the Planck length $l_P = ct_P \sim 10^{-33}cm$, and the Planck energy $E_P = \hbar/t_P \sim 10^{28}eV$. Clearly the Planck time is so short, the Planck length so minuscule, and the Planck energy so high (in elementary particle physics) that these units are not used casually. So it takes a certain amount of foolhardiness to even mention Planck-scale physics. Indeed, when Giovanni Amelino-Camelia and the author dared to give talks on this subject in the Huntsville Workshop 2002, we were duly recognized and jointly honored with an award for Physics Exotica by the Executive Organizing Committee of the Workshop.

Exotic though it may well be, Planck-scale physics has, in recent years, been garnering wider acceptance in the theoretical physics circle. The reason is clear: it is generally believed that at Planck scale, the quantum aspects of gravity become manifest. Only when we understand spacetime at Planck scale can we properly synthesize quantum mechanics with general relativity to find the correct theory of quantum gravity. By now there are quite a few approaches to quantum gravity. In addition to the front runners, string/M theory[1] and loop quantum gravity[2], the list includes causal sets, dynamical triangulations, causal dynamical triangulations, twistor theory, non-commutative geometry, supergravity, cellular networks, approaches based on analogies to condensed matter physics, and foamy structure of quantum spacetime[3]. For an assessment of these major approaches to quantum gravity (and references), see Ref.[4]. But in spite of all the impressive progress these candidate quantum gravity theories have made, it is probably fair to say that a complete and satisfactory formulation of the correct theory of quantum gravity is still not yet at hand.

Lacking such a formulation, one can hardly speak of Planck-scale physics with great confidence. But by extrapolating the well-known successes of quantum mechanics and general relativity in low energy, we believe one can still make predictions about certain phenomena involving Planck-scale physics, and check for consistency. The scope of this Brief Review is

quite limited. It concerns phenomena at an energy scale much below the Planck energy E_P. It deals only with a few topics in Planck-scale/quantum-gravity physics, or, more correctly, in the interplay between quantum mechanics and general relativity — topics with which the author has some familiarity. The focus will mostly be on the uncertainties in some observable quantities induced by the synthesis of quantum mechanics and general relativity. Our approach is very conservative. We make no assumption on the high energy regime of the ultimate quantum gravity theory, and refrain from speculating on violations of Lorentz invariance and systematically modified dispersion relations[1] which many people believe are unavoidably induced by quantum gravity. We want to see how far we can go without making those assumptions, sensible as they may well be.

The outline of this Brief Review is as follows: In section II, we introduce the subject by considering the accuracy with which one can measure a distance or a time interval. Consistent with our limited objective, the distances and the time intervals considered are understood to be much larger than l_P and t_P respectively. The problem is tackled from two different angles, by first using a gedanken experiment of spacetime measurements, and then by using the holographic principle. We interpret the resulting uncertainties in spacetime measurements as due to quantum fluctuations of spacetime, i.e., the uncertainties in distance/time measurements are due to fluctuations of the spacetime metric which, following Wheeler, we will loosely call the quantum foam or spacetime foam. Consistency between the quantum foam picture described in section II and black hole physics is considered in section III which also gives a discussion of some connections to limitations to computation and to the accuracy of a simple clock due to the fuzziness of spacetime . Fluctuations due to quantum foam are very minuscule, so they can be detected only if there is a huge cumulative effect from "summing" up the individual fluctuations. In the section IV we consider the cumulative effects of quantum foam. In section V, we consider how spacetime fluctuations induce an uncertainty in energy-momentum measurements and possibly modify dispersion relations. We also discuss how, in principle, the speed of light can be used to test the modified dispersion relation and probe Planck-scale physics. The next three sections are devoted to possible ways to do Planck-scale phenomenology. The trick is to find measurements which

[1] By this, we mean dispersion relations modified by a term with a coefficient of *fixed* magnitude and sign. More on this near the end of section VIII.

will "amplify" small effects of quantum gravity. In section VI, we consider the possibility of using spacetime foam-induced phase incoherence of light from distant galaxies to probe Planck-scale physics (here the ratio of the distance to the galaxies and the wavelength of light plays the role of the amplifying factor). In section VII we discuss the the possibility of measuring the foaminess of spacetime with interferometers (here the existence of another length scale, in addition to the Planck length, provided by the frequency of the noise spectrum plays an important role). In section VIII we entertain the idea that energy-momentum uncertainties may be the origin of threshold anomalies in ultra-high energy cosmic ray and TeV-γ events (here the amplifying factor is due to the huge discrepancy between the energies of the two colliding particles). We also discuss an alternative proposal involving "systematic" deformations of particle dispersion relations. Concluding remarks are found in the section IX which also contains a brief (and necessarily very incomplete) survey of other proposals to probe Planck-scale physics experimentally. Two related topics in Planck-scale physics are relegated to appendices. If spacetime is really discrete at the Planck scale, as suggested by some approaches to quantum gravity, then one should replace differential equations describing fundamental interactions by difference equations; in Appendix A, we derive the Dirac equation in the form of a difference equation. In Appendix B, we speculate on a possible infrared and ultravolet connection, as a result of an interplay between Planck-scale and Hubble-scale physics, encoded in the cosmological constant.

II. QUANTUM FLUCTUATIONS OF SPACETIME

At small scales, spacetime is fuzzy and foamy due to quantum fluctuations. One manifestion of the fluctuations is in the induced uncertainties in any distance measurement. We will derive the uncertainties or fluctuations by two independent methods.[5, 6] Neither method by itself is satisfactorily rigorous, but the fact that they both yield the same result bodes well for the robustness of the conclusion. Let us first consider a gedanken experiment to measure the distance l between two points. Following Wigner[7], we can put a clock at one of the points and a mirror at the other. By sending a light signal from the clock to the mirror in a timing experiment, we can determine the distance. However, the quantum uncertainty in the positions of the clock and the mirror introduces an inaccuracy δl in the distance measurement. Let us concentrate on the clock and denote its mass by m. Wigner

argued that if it has a linear spread δl when the light signal leaves the clock, then its position spread grows to $\delta l + \hbar l (mc\delta l)^{-1}$ when the light signal returns to the clock, with the minimum at $\delta l = (\hbar l/mc)^{1/2}$. Hence one concludes that

$$\delta l^2 \gtrsim \frac{\hbar l}{mc}. \tag{1}$$

One can supplement this quantum mechanical relation with a limit from general relativity[8]. To see this, let the clock be a light-clock consisting of two parallel mirrors (each of mass $m/2$), a distance d apart, between which bounces a beam of light. For the uncertainty in distance measurement not to be greater than δl, the clock must tick off time fast enough that $d/c \lesssim \delta l/c$. But d, the size of the clock, must be larger than the Schwarzschild radius Gm/c^2 of the mirrors, for otherwise one cannot read the time registered on the clock. From these two requirements, it follows that

$$\delta l \gtrsim \frac{Gm}{c^2}, \tag{2}$$

the product of which with Eq. (1) yields

$$\delta l \gtrsim (l l_P^2)^{1/3} = l_P \left(\frac{l}{l_P}\right)^{1/3}, \tag{3}$$

where $l_P = (\hbar G/c^3)^{1/2}$ is the Planck length. [2] [3]

A gedanken experiment to measure a time interval T gives an analogous expression:

$$\delta T \gtrsim (T t_P^2)^{1/3}. \tag{4}$$

The spacetime fluctuation translates into a metric fluctuation over a distance l and a time interval T given by

$$\delta g_{\mu\nu} \gtrsim (l_P/l)^{2/3}, \;\; (t_P/T)^{2/3}, \tag{5}$$

respectively.

One can also derive the δl result by appealing to the holographic principle[10] which states that the maximum number of degrees of freedom that can be put into a region of space is given by the area of the region in Planck units. Consider a region of space measuring $l \times l \times l$,

[2] Note that δl depends on both l_P and l; it is of mesoscopic nature. This mesoscopic scale may best be explored via stochastic semiclassical gravity[9].

[3] We also note that, for (N + 1)-dimensional spacetime, Eq. (3) is generalized to read $\delta l \gtrsim l_P (l/l_P)^{1/N}$.

and imgine partitioning it into cubes as small as physical laws allow. With each small cube we associate one degree of freedom. If the smallest uncertainty in measuring a distance l is δl, in other words, if the fluctuation in distance l is δl, then the smallest such cubes have volume $(\delta l)^3$. (Otherwise, one could divide l into units each measuring less than δl, and by counting the number of such units in l, one would be able to measure l to within an uncertainty smaller than δl.) Thus the maximum number of degrees of freedom, given by the number of small cubes we can put into the region of space, is $(l/\delta l)^3$. It follows from the holographic principle that $(l/\delta l)^3 \lesssim (l/l_P)^2$, which yields precisely the same expression for spacetime fluctuation δl given by Eq. (3). In fact one can reverse the argument and argue that the holographic principle has its origin in the quantum fluctuations of spacetime.[5, 6] Since the holographic principle is deeply rooted in black hole physics, this way of deriving spacetime fluctuations is highly suggestive of the deep connection between quantum foam and black hole physics.

III. CLOCKS, COMPUTATION, BLACK HOLES, AND PLANCK-SCALE PHYSICS

It is interesting that an argument, very similar to that used in the last section to deduce the structure of spacetime foam, can be applied to discuss the precision and the lifetime of a clock.[11] For a simple clock[4] of mass m, if the smallest time interval that it is capable of resolving is t and its total running time is T, one finds[11]

$$t^2 \gtrsim \frac{\hbar T}{mc^2}, \quad t \gtrsim \frac{Gm}{c^3}, \tag{6}$$

the analogue of Eq. (1) and Eq. (2) respectively. One can combine these two expressions to give

$$T/t^3 \lesssim t_P^{-2} = \frac{c^5}{\hbar G}, \tag{7}$$

which relates clock precision to its lifetime. (Note that this new expression is just Eq. (4) with t playing the role of δT.) For example, a femtosecond (10^{-15} sec) precision yields the bound $T \lesssim 10^{34}$ years. However, note that the bound on T goes down rapidly as t^3.

[4] A clock is not simple if it is made up of components which are not used to keep time concurrently.

One can translate the above clock relations into useful expressions for a simple computer. The fastest possible processing frequency is obviously given by t^{-1}. Thus we identify $\nu = t^{-1}$ as the clock rate of the computer, i.e., the number of operations per bit per unit time.[5] The identification of the number I of bits of information in the memory space of a simple computer is subtler. Since T/t is the maximum number of steps of information processing, we make the identification $I = T/t$. Using the clock relation in Eq. (7) and the identifications of ν and I in terms of t and T, one gets

$$I\nu^2 \lesssim \frac{c^5}{\hbar G} \sim 10^{86}/sec^2. \tag{8}$$

This expression links together our concepts of information, gravity, and quantum uncertainty.[11] We will see below that nature seems to respect this bound which, in particular, is saturated for black holes. For comparison, current laptops perform about 10^{10} operations per sec on 10^{10} bits, yielding $I\nu^2 \sim 10^{10}/sec^2$.

Next let us apply the two (in-)equalities in Eq. (6) to a black hole of mass m, used as a clock. It is reasonable to use the light travel time across the black hole's horizon as the resolution time of the clock, i.e., $t \sim \frac{Gm}{c^3} \equiv t_{BH}$, then one immediately finds that

$$T \sim \frac{G^2 m^3}{\hbar c^4} \equiv T_{BH}, \tag{9}$$

which is just Hawking's black hole lifetime! Thus, if we had not known of black hole evaporation, this remarkable result would have implied that there is a maximum lifetime (of this magnitude) for a black hole.[11, 12] This is another demonstration of the intimate (if, in this case, indirect) relationship between quantum foam and black hole physics.

Now in principle, it is possible to program black holes to do computations in such a way that the results of the computation can be read out of the fluctuations in the apparently thermal Hawking radiation, if black holes indeed evolve in a unitary fashion as we believe.[13] So imagine that we form a black hole (of mass m) whose initial conditions encode certain information to be processed. Then the memory space of the black hole computer has $I = T_{BH}/t_{BH} \sim (m/m_P)^2$. This gives the number of bits I as the event horizon area in Planck units, as expected from the identification[10] of black hole entropies! Furthermore, the number of operations per unit time for a black hole computer is given by $I\nu \sim mc^2/\hbar$, in agreement with Lloyd's results[13] for the ultimate physical limits to computation.

[5] This corrects a misidentification of ν in Ref.[11].

All these results indicate the conceptual interconnections of the physics underlying simple clocks, simple computers[6], black holes, and spacetime foam. We further note that, for black holes, the bounds in Eqs. (7) and (8) are saturated. Thus one can even claim that black holes are the ultimate simple clocks, and once they are programmed to do computations, they are the ultimate computers. It is curious that although they can be very massive and large, black holes are basically *simple* — a fact further supported by the no-hair theorem. Lastly we want to point out that the connections between Planck-scale physics and black hole physics discussed in this section are not unexpected in view of the consistency between spacetime measurements and the holographic principle discussed in the last section.

IV. CUMULATIVE EFFECTS OF SPACETIME FLUCTUATIONS

So far we have been discussing a particular spacetime foam model, motivated by an elementary consideration of distance measurements and by the consistency with the holographic principle. Let us now introduce a parameter $\alpha \sim 1$ to specify the different quantum foam models (also called quantum gravity models). In terms of the parameter α, the distance and metric uncertainties/fluctuations take the form

$$\delta l \gtrsim l \left(\frac{l_P}{l}\right)^{\alpha}, \quad \delta g_{\mu\nu} \gtrsim (l_P/l)^{\alpha}. \tag{10}$$

The standard choice[15] of α is $\alpha = 1$; the choice of $\alpha = 2/3$ discussed above appears[5, 11] to be consistent with the holographic principle and black hole physics and will be called the holography model; $\alpha = 1/2$ corresponds to the random-walk model found in the literature[16, 17]. Though much of our discussion below is applicable to the general case, we will use these three cases as examples of the quantum gravity models. Note that all the three quantum gravity models predict a very small distance uncertainty: e.g., even on the size of the whole observable universe ($\sim 10^{10}$ ligh-years), Eq. (10) yields a fluctuation of only about 10^{-33} cm, 10^{-13} cm, and 10^{-2} cm for $\alpha = 1, 2/3$ and $1/2$ respectively.

Let us now examine the cumulative effects[18] of spacetime fluctuations over a large distance. Consider a distance L (which will denote the distance between extragalactic sources and the telescope in section VI), and divide it into L/λ equal parts each of which has length λ

[6] For a quantum computer view of spacetime at the Planck scale, see Ref.[14]

(which, for the discussion in section VI, will *naturally* denote the wavelength of the observed light from the distant source). If we start with $\delta\lambda$ from each part, the question is how do the L/λ parts add up to δL for the whole distance L. In other words, we want to find the cumulative factor $\mathcal{C}_\alpha$ defined by

$$\delta L = \mathcal{C}_\alpha \, \delta\lambda, \tag{11}$$

Since $\delta L \sim l_P(L/l_P)^{1-\alpha}$ and $\delta\lambda \sim l_P(\lambda/l_P)^{1-\alpha}$, the result is

$$\mathcal{C}_\alpha = \left(\frac{L}{\lambda}\right)^{1-\alpha}, \tag{12}$$

in particular,

$$\mathcal{C}_{\alpha=1/2} = (L/\lambda)^{1/2}, \;\; \mathcal{C}_{\alpha=2/3} = (L/\lambda)^{1/3}, \;\; \mathcal{C}_{\alpha=1} = (L/\lambda)^0 = 1, \tag{13}$$

for the random walk $\alpha = 1/2$ case, the holography $\alpha = 2/3$ case and the "standard" $\alpha = 1$ case respectively. Note that $\mathcal{C}_{\alpha=1} = 1$ is *independent* of L. Strange as it may seem, the result is not unreasonable if we recall, for the "standard" model, $\delta l \gtrsim l_P$, independent of l. The crucial point to remember is that, for all the quantum gravity models, *none* of the cumulative factors is linear in (L/λ), i.e.,

$$\frac{\delta L}{\delta\lambda} \neq \frac{L}{\lambda}. \tag{14}$$

The reason for this is obvious: the $\delta\lambda$'s from the L/λ parts in L do *not* add coherently.[7] In fact, according to Eq. (12), the cumulative effects are linear in L/λ only for the physically unacceptable case of $\alpha = 0$ for which $\delta l \sim l$. To obtain the correct cumulative factor (given by Eq. (12)) from what we may inadvertently think it is, viz., (L/λ) (independent of α), we have to put in the *correction factor* $(\lambda/L)^\alpha$.

[7] To gain insight into the process, consider the $\alpha = 1/2$ random-walk model of quantum gravity, and for simplicity, assume that $\delta\lambda$ takes on only two values, viz. $\pm l_P(\lambda/l_P)^{1/2}$, with equal probability (instead of, say, a Gaussian distribution about zero, which is more likely). If the fluctuations from the different segments are all of the same sign, then together they contribute $\pm l_P(\lambda/l_P)^{1/2} \times (L/\lambda)$ to δL. But both these two cases, yielding a linear L-dependence for δL, are extremely unlikely (each having a probablity of $1/2^{L/\lambda} \ll 1$ for $(L/\lambda) \gg 1$.) For this one-dimensional random walk involving L/λ steps of equal size $(\delta\lambda)$, each step moving right or left (corresponding to $+$ or $-$ sign) with equal probability, the result is well-known: the cumulative fluctuation is given by $\delta L \sim \delta\lambda \times (L/\lambda)^{1/2}$ which is $l_P(L/l_P)^{1/2}$ as expected for consistency.

V. ENERGY-MOMENTUM UNCERTAINTIES

Just as there are uncertainties in distance and time interval measurements, there are uncertainties in energy-momentum measurements. Both types of uncertainties[8, 19] come from the same source, viz., quantum fluctuations of space-time metrics[20] giving rise to space-time foam. Imagine sending a particle of momentum p to probe a certain structure of spatial extent l so that $p \sim \frac{\hbar}{l}$. Consider the coupling of the metric to the energy-momentum tensor of the particle, $(g_{\mu\nu} + \delta g_{\mu\nu})t^{\mu\nu} = g_{\mu\nu}(t^{\mu\nu} + \delta t^{\mu\nu})$, where we have noted that the uncertainty in $g_{\mu\nu}$ can be translated into an uncertainty in $t_{\mu\nu}$. Eq. (10) for $\delta g_{\mu\nu}$ can now be used to give

$$\delta p = \beta p \left(\frac{p}{m_P c}\right)^{\alpha}, \tag{15}$$

where $\beta \sim 1$ and m_P is the Planck mass.[8] Another way to derive the momentum uncertainty is to regard δp as the uncertainty of the momentum operator $p = -i\hbar\partial/\partial x$, associated with $\delta x = x(l_P/x)^{\alpha}$.[8, 19] The corresponding statement for energy uncertainties is

$$\delta E = \gamma E \left(\frac{E}{E_P}\right)^{\alpha}, \tag{16}$$

with $\gamma \sim 1$. Note that the energy-momentum uncertainty is actually fixed by dimensional analysis, once the uncertainty in the metric is given by Eq. (10). We emphasize that all the uncertainties take on $\pm$ sign with equal probability (most likely, a Gaussian distribution about zero).

What is the time scale at which these fluctuations occur? Returning to the gedanken experiment in distance l measurement discussed in section II, we may be tempted to conclude that the δl fluctuation occurs at a time scale given by l/c, independent of the Planck-scale. Accordingly, the energy fluctuation δE occurs at a time scale $\hbar/E$. However, this argument is by no means convincing. While it does take $\sim l/c$ amount to time to make one measurement of the distance, to measure the fluctuations of the distance we need to take more than one measurement. We are led to ask how quickly we can make a succession of measurements of the distance and perhaps then we would argue that it is the time separation between the different measurements that is the relevent time scale at which the fluctuations occur. The question of which is the correct time scale to use has yet to be settled.

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[8] Alternatively, one can simply use $p \sim \frac{\hbar}{l}$ (so that $\delta p \sim (\hbar/l^2)\delta l$) in conjunction with $\delta l \gtrsim l(l_P/l)^{\alpha}$.

Energy-momentum uncertainties affect both the energy-momentum conservation laws and the dispersion relations. Energy-momentum is conserved up to energy-momentum uncertainties due to quantum foam effects, i.e., $\Sigma(p_i^\mu + \delta p_i^\mu)$ is conserved with p_i^μ being the average values. On the other hand the dispersion relation is now generalized to read

$$E^2 - p^2 - \epsilon p^2 \left(\frac{p}{E_P}\right)^\alpha = m^2, \tag{17}$$

for high energies with $E \gg m$. Here and hereafter, unless clarity demands otherwise, we set the speed of light c, as well as Planck's constant $\hbar$, equal to unity. For $\epsilon \neq 0$, one can interpret the additonal term in the dispersion relation as an additional contribution to the mass which is energy-dependent. *A priori* we expect $\epsilon \sim 1$ and is independent of β and γ. But due to our ignorance of quantum gravity, we cannot make any definite statements. It is possible that $\epsilon \approx 2(\beta - \gamma)$, which would be the case if the dispersion relation is given by $(E+\delta E)^2 - (p+\delta p)^2 = m^2$. Another possibility is that $\epsilon \approx 0$, which would be the case if the usual dispersion relation holds for the average E and p. One can reach the latter conclusion if one appeals to the van-Dam-Veltman-Zakharov discontinuity theorem[21] which states that the theory for an exactly massless graviton is different from that for an extremely light graviton. If graviton is indeed the quantum mediator of gravitational interactions (in a Minkowskian spacetime), according to Ref.[21], only the theory for an exactly massless graviton can explain Einstein's three tests of general relativity. To the extent that the vDVZ theorem is correct, one infers that the dispersion relation for gravitons is unaffected by quantum fluctuations of spacetime, i.e., $\epsilon = 0$, so that the graviton does not have an energy-dependent effective mass and remains exactly massless. Then it is not hard to imagine that the dispersion relation for other particles, especially the massless particles like the photon, is also not affected.

But if $\epsilon \neq 0$, the modified dispersion relation discussed above has an interesting consequence for the speed of light. Applying Eq. (17) to the massless photon yields

$$E^2 \simeq c^2 p^2 + \epsilon E^2 \left(\frac{E}{E_P}\right)^\alpha, \tag{18}$$

where we have restored the factor of c . The speed of (massless) photon

$$v = \frac{\partial E}{\partial p} \simeq c\left(1 + \epsilon \frac{1+\alpha}{2}\frac{E^\alpha}{E_P^\alpha}\right), \tag{19}$$

becomes energy-dependent if $\epsilon \neq 0$, and it fluctuates around c. This fluctuating speed of light would *seem* to yield[22] an energy-dependent spread in the arrival times of photons of the

same energy E given by $\delta t \sim |\epsilon| t (E/E_P)^\alpha$, where t is the average overall time of travel from the photon source, say, a gamma ray burster, for which both t and E are relatively large. Furthermore, the modified energy-momentum dispersion relation would seem to predict time-of-flight differences between simultaneously-emitted photons of different energies, E_1 and E_2, given by

$$\delta t \simeq \epsilon t \frac{1+\alpha}{2} \frac{E_1^\alpha - E_2^\alpha}{E_P^\alpha}. \tag{20}$$

An upper bound[23, 24] on the absolute value of ϵ could then be obtained from the observation[25] of simultaneous (within experimental uncertainty of $\delta t \leq 200$ sec) arrival of 1-TeV and 2-TeV γ-rays from Mk 421 which is believed to be ~ 143 Mpc away from the Earth. But these results for the spread of arrival times of photons are *not* correct, because we have inadvertently forgotten to put in the necessary correction factors discussed in the last section. For the spread in arrival time of the photons of the same energy, taking into account the correction factor $(\lambda/L)^\alpha \sim (\hbar/Et)^\alpha$, we get a much smaller $\delta t \sim t^{1-\alpha} t_P^\alpha$. We will witness the importance of the correction factor again in the next section. But note that, if ϵ is constant (instead of fluctuating about zero), then Eq. (20) does give the correct time-of-flight differences. See sections VIII and IX.

VI. INDUCED PHASE INCOHERENCE OF LIGHT FROM GALAXIES

Recently Lieu and Hillman[26, 27] and then Ragazzoni, Turatto, and Gaessler [28] proposed a technique that has hitherto been overlooked to directly test the Planck scale fluctuations. They argued that these fluctuations can cumulatively lead to a complete loss of phase for radiations that have propagated a sufficiently large distance and they searched for patterns of images of very distant galaxies gathered by telescopes that should not be present if prevailing notions of spacetime quantum is correct. In this section, we[18] critically examine their very interesting idea.

Consider the phase behavior of light with wavelength λ received from a celestial optical source located at a distance L away. During the propagation time $T = L/v_g$ where v_g is the group velocity of propagation, the phase has advanced by the amount

$$\phi = 2\pi \frac{v_p T}{\lambda} = 2\pi \frac{v_p}{v_g} \frac{L}{\lambda}, \tag{21}$$

where v_p is the phase velocity of the light wave. This phase fluctuates randomly according to

$$\begin{aligned}\delta\phi &= 2\pi\frac{L}{\lambda}\,\delta\!\left(\frac{v_p}{v_g}\right) + 2\pi\frac{v_p}{v_g}L\,\delta\!\left(\frac{1}{\lambda}\right) + 2\pi\frac{v_p}{v_g}\,\frac{1}{\lambda}\,\delta L \\ &= \delta\phi_1 + \delta\phi_2 + \delta\phi_3, \end{aligned} \tag{22}$$

with $\delta\phi_i$ denoting the three successive terms in $\delta\phi$ ($i = 1, 2, 3$). Note that the various expressions for $\delta\phi_i$ have been obtained by simple-minded straightforward algebra. The quantum gravitational and statistical nature of the fluctuations have yet to be properly incorporated, i.e., we still have to fold in the correction factors for the cumulative effects, as discussed in section IV.

For $\delta\phi_1$, applying Eq. (17) with $m = 0$ for photon, and recalling that $v_p = E/p$ and $v_g = dE/dp$, we obtain

$$\delta\left(\frac{v_p}{v_g}\right) \sim \left(\frac{E}{E_P}\right)^{\alpha} = \left(\frac{l_P}{\lambda}\right)^{\alpha}, \tag{23}$$

where we have used $E/E_P = l_P/\lambda$ and $\epsilon \sim 1$. Putting in the correction factor $(\lambda/L)^{\alpha}$, we obtain

$$\delta\phi_1 \sim 2\pi\frac{L}{\lambda}\left(\frac{l_P}{\lambda}\right)^{\alpha}\left(\frac{\lambda}{L}\right)^{\alpha} = 2\pi\frac{l_P^{\alpha}L^{1-\alpha}}{\lambda}. \tag{24}$$

For $\delta\phi_2$, it suffices to approximate v_p/v_g by 1. Recalling that $\delta\lambda \sim l_P(\lambda/l_P)^{1-\alpha}$ and the need to put in the correction factor $(\lambda/L)^{\alpha}$, we get $\delta\phi_2 \sim \delta\phi_1$. For $\delta\phi_3$, also approximating v_p/v_g by 1, using $\delta L \sim l_P(L/l_P)^{1-\alpha}$, and noting that there is no need for a correction factor for this term, we immediately find $\delta\phi_3 \sim \delta\phi_1$. Since all the three $\delta\phi_i$'s are of the same order of magnitude, we conclude that

$$\delta\phi = 2\pi a\frac{l_P^{\alpha}L^{1-\alpha}}{\lambda} \sim 2\pi\frac{\delta L}{\lambda}, \tag{25}$$

where $a \sim 1$. In passing, we note that, since $\delta\phi_1$ involves energy-momentum fluctuations whereas both $\delta\phi_2$ and $\delta\phi_3$ involve distance fluctuations, the fact that they all make contributions of the same order of magnitude can be taken as a sign of consistency between Eq. (10), Eq. (15) and Eq. (16).

In stellar interferometry, following Lieu and Hillman's[26, 27] reasoning, for light waves from an astronomical source incident upon two reflectors (within a terrestrial telescope) to subsequently converge to form interference fringes, it is necessary that $\delta\phi \lesssim 2\pi$. But the analysis of the principles of interferometry of distant *incoherent* astronomical "point" sources

can be tricky. The local spatial coherence across an interferometer's aperature for photons from a distant point source (i.e., plane waves) is a reflection of the fact that all photons have the same resultant phase differences *across the interferometer*. However, as Lieu and Hillman pointed out, this local coherence can be lost if there is an intervening medium such as a turbulent plasma or spacetime foam capable of introducing small changes into the "effective" phases of the photon stream falling on the interferometer. Such spacetime foam-induced phase differences are themselves incoherent and therefore must be treated with the *correct cumulative factors* $\mathcal{C}_\alpha$ appropriate for the quantum gravity model under consideration.

First we note that since the cumulative factor for the "standard" model of quantum gravity (for which $\alpha = 1$) is 1, i.e., there is no cumulative effect, obviously Lieu and Hillman's proposed approach cannot be used to rule out (or confirm) the $\alpha = 1$ model. To rule out a certain model with $\alpha < 1$, the strategy is to look for unexpected interference fringes for which the phase coherence of light from the distant sources should have been lost (i.e., $\delta\phi \gtrsim 2\pi$) for that value of α according to theoretical calculations. Consider the example cited by Lieu and Hillman[27], involving the clearly visible Airy rings in an observation of the active galaxy PKS1413+135 ($L = 1.216$ Gpc) by the HST at $\lambda = 1.6\mu m$ wavelength[29]. For this example, Eq. (25) yields $\delta\phi \sim 10 \times 2\pi a$ for the random walk $\alpha = 1/2$ model and $\delta\phi \sim 10^{-9} \times 2\pi a$ for the holography $\alpha = 2/3$ model. Since we expect $a \sim 1$, the observation of Airy rings in this case would seem to marginally, if at all, rule out the random walk model. On the other hand, the holography model is obviously not ruled out. This finding contradicts the conclusion reached recently by Lieu and Hillman[27] who argued that the HST detection of Airy rings from PKS1413+135 has ruled out a majority of modern models of quantum gravity, including the "standard" $\alpha = 1$ model. (Earlier, Lieu and Hillman[26] had claimed to have ruled out the $\alpha = 2/3$ model by noticing that interference effects were clearly seen in the Infra-red Optical Telescope Array[30] at $\lambda = 2.2\mu m$ light from the star S Ser which is ~ 1 kpc away.) The resolution of this disagreement lies in the fact that Lieu and Hillman neglected to take into account the correction factor in estimating the cumulative effects of spacetime foam. This neglect resulted in their overestimate of the cumulative effects by a factor $(L/\lambda)^\alpha$: for the case of PKS1413+135, by 10^{20} and 10^{30} for $\alpha = 2/3, 1$ respectively. Subsequent work by Ragazzoni et al.[28] contains the same error of assuming that the cumulative factor is (L/λ) rather than the correct factor $(L/\lambda)^{1-\alpha}$. Their claim

that the $\alpha = 2/3$ model and the $\alpha = 1$ model are ruled out is also far from being justified. We note that Coule[31] has independently pointed out that "Planck scale is still safe from stellar images" using another argument.

VII. DETECTING QUANTUM FOAM WITH INTERFEROMETERS

As pointed out recently [5, 16], modern gravitational-wave interferometers, through future refinements, may reach displacement noise level low enough to test a subset of the space-time foam models. To see this, in any distance measurement that involves a time interval τ, we note that there is a minute uncertainty

$$\sigma \sim l_P^\alpha (c\tau)^{1-\alpha}. \tag{26}$$

This uncertainty manifests itself as a displacement noise (in addition to other sources of noises) that infests the interferometers. Modern gravitational-wave interferometers are sensitive to changes in distances to an accuracy of $\sim 10^{-18}$ m or better. True, this extraordinary sensitivity is still no where near the Planck length. But what really counts is whether the length scale characteristic of the associated noise of quantum foam at the frequency of the interferometer bandwidth is comparable to the sensitivity level of the interferometer. For an interferometer with bandwidth centered at frequency f, the relevant length scale characteristic of the noise due to spacetime foam is given by $l_P^\alpha (c/f)^{1-\alpha}$. Interestingly, within certain range of frequencies, the experimental limits are comparable to the theoretical predictions for some of the quantum gravity models.

One can analyse the displacement noise in terms of the associated displacement amplitude spectral density $S(f)$ of frequency f. For a frequency-band limited from below by the time of observation t, σ is given in terms of $S(f)$ by[32]

$$\sigma^2 = \int_{1/t}^{f_{max}} [S(f)]^2 df. \tag{27}$$

Now we can easily check that, for the displacement noise given by Eq. (26), the associated $S(f)$ is

$$S(f) \sim c^{1-\alpha} l_P^\alpha f^{\alpha - \frac{3}{2}}. \tag{28}$$

By comparing the spectral density with the existing observed noise level[33] of $3 \times 10^{-17}\mathrm{cm} - \mathrm{Hz}^{-1/2}$ near 450 Hz, the lowest noise level reached by the Caltech 40-meter in-

terferometer, we obtain the bound $l_P \lesssim 10^{-15}, 10^{-27}$ and 10^{-38} cm for the quantum gravity models given by $\alpha = 1, 2/3$ and $1/2$ respectively. The "advanced phase" of LIGO[34] is expected to achieve a displacement noise level of less than 10^{-20}mHz$^{-1/2}$ near 100 Hz, and this would probe l_P down to $10^{-17}, 10^{-31}$ and 10^{-43} cm for $\alpha = 1, 2/3$ and $1/2$ respectively. This analysis seems to suggest that the random walk $\alpha = 1/2$ model is already ruled out. But more excitedly, modern gravitational-wave interferometers appear to be within striking distance of testing the holography ($\alpha = 2/3$) quantum gravity model. Since $S(f) \sim f^{-5/6}$ for this model (see Eq. (28)), we can optimize the performance of an interferometer at low frequencies. As lower frequency detection is possible only in space, one might think that the planned LISA[35] is more suitable for our purpose. Unfortunately, LISA loses more due to its greater arm length than what it gains by going to lower frequencies.[9]

We also note that the correlation length of quantum gravity fluctuation noise is extremely short as the characteristic scale is the Planck length. Thus it can easily be distinguished from other noises because of this lack of correlation. In this regard, it may be useful for quantum gravity studies to have two nearby interferometers.

So far we have concentrated on the observation along the propagation direction of light in the interferometer. A matter of concern is the effect of the beam size in the transverse direction.[10] Implicit in the discussion above is the assumption that spacetime in between the mirrors in the interferometer fluctuates coherently for all the photons in the beam. But the large beam size in LIGO (compared to the Planck scale) makes such coherence unlikely. Thus a small beam interferometer of comparable power and phase sensibility would definitely be much more sensitive to the predicted effects of quantum gravity. Obviously building such a dedicated interferometer would be very valuable. But we cannot emphasize enough the importance of LIGO achieving its best noise limit which, even in the form of negative results, will still be of utmost interests to us.

Finally we mention that there have been suggestions to use atom interferometers and optical interferometers [6, 37, 38, 39] (with minimal beam size effects) to look for effects of spacetime fluctuations.

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[9] As noted by Amelino-Camelia [36], the relevant quantity is the strain noise power spectrum which is given by displacement spectral density divided by the square of the arm length.

[10] This part of the discussion is based on private communications with G. Amelino-Camelia and R. Weiss.

VIII. ENERGY-MOMENTUM UNCERTAINTIES AND THE UHECR EVENTS

The universe appears to be more transparent to the ultra-high energy cosmic rays (UHECRs)[40] and multi-TeV γ-rays[41] than expected. Theoretically one expects the UHECRs to interact with the Cosmic Microwave Background Radiation and produce pions, and the TeV photons to interact with the Far Infra Red Background (FIRB) photons and produce electron-positron pairs. These interactions above the respective threshold energies should make observations of UHECRs with $E > 5{\cdot}10^{19}$eV (the GZK limit)[42] or of gamma-rays with $E > 10$TeV[43] from distant sources unlikely. Still UHECRs above the GZK limit and, with much less data and some uncertainties about the FIRB, Mk501 photons with energies up to 24 TeV have been observed. In this section, we attempt to explain the (well-established) UHECR paradox and the (not so-well-established) TeV-γ puzzle, by arguing[22] that energy-momentum uncertainties due to quantum gravity (significant only for high energy particles like the UHECRs and TeV-γ-rays), too small to be detected in low-energy regime, can affect particle kinematics so as to raise or even eliminate the energy thresholds, thereby explaining the threshold anomalies, in these two reactions.[11] (For similar or related approaches, see Ref.[44].)

Relevant to the discussion of the UHECR events and the TeV-γ events is the scattering process in which an energetic particle of energy E_1 and momentum $\mathbf{p}_1$ collides head-on with a soft photon of energy ω in the production of two energetic particles with energy E_2, E_3 and momentum $\mathbf{p}_2$, $\mathbf{p}_3$. After taking into account energy-momentum uncertainties, the conservation laws demand

$$E_1 + \delta E_1 + \omega = E_2 + \delta E_2 + E_3 + \delta E_3, \tag{29}$$

and

$$p_1 + \delta p_1 - \omega = p_2 + \delta p_2 + p_3 + \delta p_3, \tag{30}$$

where δE_i and δp_i ($i = 1, 2, 3$) are given by Eqs. (16) and (15),

$$\delta E_i = \gamma_i E_i \left(\frac{E_i}{E_P}\right)^{\alpha}, \quad \delta p_i = \beta_i p_i \left(\frac{p_i}{m_P c}\right)^{\alpha}, \tag{31}$$

[11] Unfortunately, we have nothing useful to say about the origins of these energetic particles per se.

and we have omitted $\delta\omega$, the contribution from the uncertainty of ω, because ω is small.[12]

Combining Eq. (31) with the modified dispersion relations[13] Eq. (17) for the incoming energetic particle ($i = 1$)and the two outgoing particles ($i = 2, 3$),

$$E_i^2 - p_i^2 - \epsilon_i p_i^2 \left(\frac{p_i}{E_P}\right)^\alpha = m_i^2, \tag{32}$$

we obtain the threshold energy equation

$$E_{th} = p_0 + \tilde{\eta}\frac{1}{4\omega}\frac{E_{th}^{2+\alpha}}{E_P^\alpha}, \tag{33}$$

where

$$p_0 \equiv \frac{(m_2 + m_3)^2 - m_1^2}{4\omega} \tag{34}$$

is the (ordinary) threshold energy if there were no energy-momentum uncertainties, and

$$\tilde{\eta} \equiv \eta_1 - \frac{\eta_2 m_2^{1+\alpha} + \eta_3 m_3^{1+\alpha}}{(m_2 + m_3)^{1+\alpha}}, \tag{35}$$

with

$$\eta_i \equiv 2\beta_i - 2\gamma_i - \epsilon_i. \tag{36}$$

Note that, in Eq. (33), the quantum gravity correction term is enhanced by the fact that ω is so small (compared to p_0).

Given that all the β_i's, the γ_i's and the ϵ_i's are of order 1 and can be $\pm$, $\tilde{\eta}$ can be $\pm$ (taking on some unknown Gaussian distribution about zero), but it cannot be much bigger than 1 in magnitude. For positive $\tilde{\eta}$, E_{th} is greater than p_0. The threshold energy increases with $\tilde{\eta}$ to $\frac{3}{2}p_0$ at $\tilde{\eta} = \tilde{\eta}_{max}$, beyond which there is no (real) physical solution to Eq. (33) (i.e., E_{th} becomes complex) and we interpret this as *evading* the threshold cut.[22] The cutoff $\tilde{\eta}_{max}$ is very small: $\tilde{\eta}_{max} \sim 10^{-14}, 10^{-17}$ for $\alpha = 1, 2/3$ respectively for UHECRs; it is more modest for TeV-γ-rays: $\tilde{\eta}_{max} \sim 1, 10^{-5}$ for $\alpha = 1, 2/3$ respectively. Thus, energy-momentum uncertainties due to quantum gravity, too small to be detected in low-energy regime, can (in principle) affect particle kinematics so as to raise or even eliminate energy thresholds. Can this be the solution to the UHECR and multi-TeV γ-ray threshold anomaly puzzles?

[12] We should mention that we have not found the proper (possibly nonlinear) transformations of the energy-momentum uncertainties between different reference frames. Therefore we apply the results only in the frame in which we do the observations.

[13] The suggestion that the dispersion relation may be modified by quantum gravity first appeared in Ref.[45].

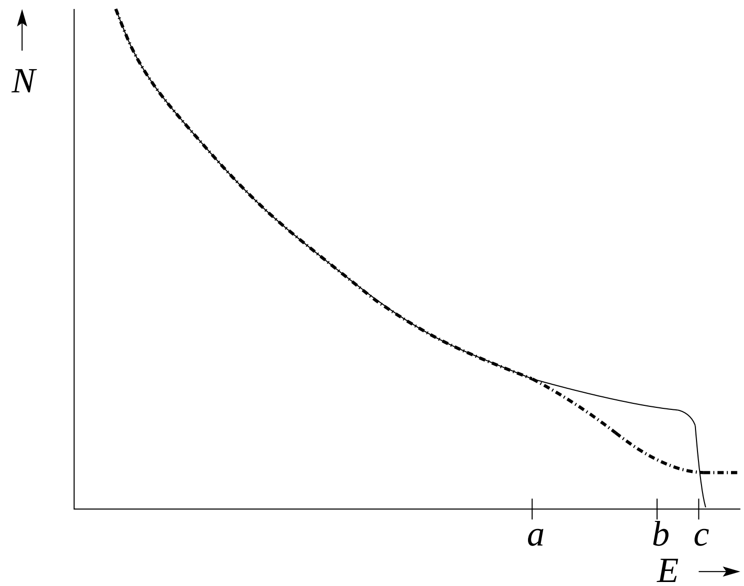

FIG. 1: Schematic plot of the number N of UHECRs versus energy E. The solid curve refers to the case of ordinary threshold energy $E_{th} = p_0$. The dashed-dotted curve refers to the case of the threshold energy given by Eq. (33). The "knee" region is indicated by "a", the "ankle" region by "b", and the GZK limit by "c".

On the other hand, for negative $\tilde{\eta}$, the threshold energy is less than p_0, i.e., a negative $\tilde{\eta}$ *lowers* the threshold energy.[46, 47, 48] For the case of multi-TeV γ-rays, $\tilde{\eta} \sim -1$ yields $E_{th} \sim 0.9p_0$ for $\alpha = 1$ model. The situation for the case of UHECRs is more interesting: $\tilde{\eta} \sim -1$ gives $E_{th} \sim 10^{15}$eV for $\alpha = 1$.[47, 48] Can this be the explanation of the opening up of the "precocious" threshold in the "knee" region?

It is far too early to call this a success. But an optimistic assessment, indicated by the schematic plot in Figure 1 of the number of UHECRs versus energy, invites one to wonder if the "ankle" region is also "explained". There is a similar plot for the TeV-γ events.

However, before we get carried away, we should be aware that there are at least two problems that confront this particular proposal to solve the two astrophysical puzzles (besides the obvious problem of making the analysis quantative). First of all, there is no guarantee that $\tilde{\eta}$ is not too small; if, e.g., $\mid \tilde{\eta} \mid \leq \tilde{\eta}_{max}$, then the problems are not ameliorated. But even if $\tilde{\eta}$ spreads over a large enough range of $\pm$ values, there is still a potentially serious barrier to overcome. Let us concentrate on the case of TeV- γ-rays. In the above discussion[46], we have analyzed a single photon-photon collision, focusing on the kinematic requirements for electron-positron pair production. For a Mk501 photon with energy of some 10 or 20

TeV, there are many opportunities to collide with soft photons with energy suitable for pair production to occur.[14] Thus one expects that even a small probability of producing an electron-positron pair in a single collision might be sufficient to lead to the disappearance of the MK501 hard photon before reaching our detectors. The probability is small in a single collision with a soft background photon, but the fact that there are, during the long journey, many such pair-production opportunities renders it likely that in one of the many collisions the hard photon would indeed disappear into an electron-positron pair. Completely analogous arguments apply to the analysis of ultra-high-energy cosmic rays. While this does not exclude altogether the idea that energy-momentum fluctuation effects might be responsible for the threshold anomalies, it does demonstrate the need for further study of this particular scenario.

So far we have considered the physics purely based on quantum-gravity-induced uncertainties. But now we realize that the potential difficulties encountered by the above proposal to solve the UHECR and multi-TeV γ-ray threshold puzzles can in fact be overcome if ϵ in the modified dispersion relation takes on a *fixed* (large enough) *negative*[15] value

$$E_i^2 - p_i^2 - \epsilon p_i^2 \left(\frac{p_i}{E_P}\right)^\alpha = m_i^2, \;\; \epsilon < 0, \tag{37}$$

and the ordinary energy-momentum conservation holds (i.e., $\beta = \gamma = 0$). Unlike the above proposal which involves fundamental uncertainties in some observable quantities like energy-momentum, this proposal[16] systematically shifts the threshold energies. In the terminology of Ref.[46], the above approach is said to involve a non-systematic effect of quantum gravity while the latter (different) approach involves a systematic quantum gravity effect. It has been argued[17] that quantum gravity-induced deviations from ordinary Lorentz invariance at Planck scale might lead to such a systematic deformation of the dispersion relation. In that case, both systematic and non-systematic effects may be present. It is therefore important that we understand how systematic effects and nonsystematic effects can combine in physical contexts such as the ones pertaining to the paradoxes we have considered.[46] A systematic

[14] The mean free path is much shorter than the distance between the source and the Earth.
[15] The positive sign is rejected as required by matter stability. More on matter (in)stability later.
[16] Chronologically, this proposal was put forth earlier. See, e.g., Ref.[24].
[17] For example, in the framework of loop quantum gravity. The "modified special relativity" or "doubly special relativity" approach[49] provides another example where the dispersion relation is systmatically modified, though the expressions for the conservation of energy-momentum are in general also modified in this approach.

effect may well raise the threshold for a particle-production process, but in the presence of an accompanying non-systematic effect, this increase of the threshold will have to be interpreted only in a statistical sense: processes with energetics below the new higher threshold can still (with however small probability) occur.

In addition to the potential difficulties discussed above, there is another potential problem for the approach involving "non-systematic" effects of quantum gravity. There is the question of matter (in)stability[50] in connection with this approach because quantum fluctuations in dispersion relations Eq. (32) can *lower* as well as raise the reaction thresholds. This problem may force us to entertain one or a combination of the following possibilities: (1) The fluctuations of the energy-momentum of a particle are not completely uncorrelated (e.g, the fluctuating coefficients β, γ, and ϵ in Eqs. (15), (16), and (17) may be related such that $\eta_i \approx 0$ in Eq. (36)); (2) The time scale at which quantum fluctuations of energy-momentum occur is relatively short [18] (compared to the relevant interaction or decay times); (3) Both "systematic" and "non-systematic" effects of quantum gravity are present, but the "systematic" effects are large enough to overwhelm the "non-systematic" effects.[19]

IX. CODA: OTHER SUGGESTIONS TO PROBE PLANCK-SCALE PHYSICS

In the preceding sections we have discussed several ways to probe Planck-scale physics experimentally. They include

1. looking for energy dependence of speed of light in timing arrival of high energy γ-rays with the *same* as well as *different* energies (section V);

2. looking for quantum foam-induced phase incoherence of light from extragalactic sources (section VI);

3. using interferometers to detect displacement noise due to spacetime fluctuations (section VII);

4. examining the UHECR and multi-TeV γ-ray events and explaining the threshold anoma-

[18] Unfortunately, these two scenarios also preclude the possibility that energy-momentum uncertainties are the origin of the threshold anomalies discussed above

[19] Some pundits may even entertain the possibility that energy-momentum fluctuations are negligible (in contrast to spacetime fluctuations), for in that case, the kinematics that allows matter instability is no longer operative.

lies (section VIII).

We have also suggested using black holes to probe Planck-scale physics[20] theoretically (section III). In principle, these are all potential tests of quantum-gravity-induced uncertainties. But, in practice, these tests are either difficult or almost impossible (as seen in the preceding sections). Improved techniques and new ideas will be the key to test such "*nonsystematic*" quantum-gravity effects (in the terminology of Ref.[46]).

There are other suggestions to probe Planck-scale physics that have appeared in the literature. Many of them provide constraints on the modified dispersion relation Eq. (37), i.e., they are proposals to test some of the purported "*systematic*" quantum-gravity effects. In the following we list some of them:

1. Neutral kaon decay[52]: Laboratory experiments may probe possible quantum nature of spacetime and possible CPT-violating effects induced by quantum gravity in the neutral kaon system.

2. Clock comparison experiments and experiments with spin-polarized matter[53]: These experiments have been used to put stringent bounds on violations of Lorentz symmetry and dispersion relations for nucleons, electrons, photons and light quarks.

3. Vacuum Cherenkov effects[54]: Absence of such radiation and photon decay can be used to constrain potential corrections to Lorentz invariance.

4. Suppression of pion decay at high energy[55]: Experimental data on the longitudinal development of the air showers produced by ultra-high energy hadronic primaries appear to require that ultra-high energy neutral pions are more stable than low energy pions as if the phase space for decay into two photons is reduced at high energy on account of the modified dispersion relation.

5. Synchrontron radiation in the Crab nebula[56]: It has been argued that observations of such radiation put extraordinarily stringent bounds on the modifications of the dispersion relations for photons and electrons, but some of the assumptions that go into the analysis have been challenged.

6. Birefringence effects[57]: Evidence of quantum gravity produced birefringence can be searched for by analyzing polarized light from distant sources.

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[20] On the other hand, black hole physics and the associated Hawking-Unruh effect may be probed experimentally via extremely violent acceleration provided by a standing-wave of intense lasers[51].

7. High energy γ rays from GRB[23, 58, 59]: By searching for different arrival times for photons in different energy ranges (recall Eq. (20)), e.g., pulses emitted by GRBs in different energy channels, one can test Poincare symmetries and the dispersion relation Eq. (37).

Nowadays no review on Planck-scale physics is complete without mentioning the so-called "doubly special relativities"[49] or "modified special relativities," proposed recently as possible Planck-scale modifications of the ordinary Lorentz group. They are related to "κ-deformed Lorentz symmetry"[60], and provide non-linear realizations of Lorentz symmetry with two fundamental invariants: the speed of light and a length scale usually taken to be the Planck length. Their geometries are non-commutative, giving rise to deformed commutative relations, and generalized uncertainty principles. A related proposal[61] based on the requirement of algebraic stability, yields a class of relativistic quantum algebras characterized by both the speed of light and a fundamental length which can be taken to be the Planck length.

We conclude with a few remarks on two applications of Planck-scale physics to cosmology. (See also Appendix B.) Planck-scale physics and cosmology are linked by the big bang theory. Due to the redshifting that occurred during inflation, wavelengths which correspond to cosmological lengths in the present era were smaller than the Planck length during the early stages of inflation. Thus Planck-scale physics probably played a crucial role in the generation of quantum modes in inflation.[62] The effects of these modes might be imprinted in the pattern of cosmological fluctuations we see in the cosmic mircrowave background and the large-scale structure today.

Transplanckian physics may also account for the dark energy observed in the present unverse. By employing a nonlinear dispersion relation to model the transplanckian regime, one can get ultralow frequencies at very high momenta (or very short distances).[63] It has been argued that the ultralow energy modes are still frozen today by the expansion of the universe. Their energy provides a good candidate for the dark energy which powers the accelerating expansion of the universe in the present era.

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Appendix A: Dirac equation on a Planck-scale spacetime lattice

If spacetime is discrete at the Planck scale, then fundamental equations applicable at short distances must be written in the form of difference equations rather than differential equations. In this appendix we present a form of Dirac equation on a "cubic" $(1+1)$-dimensional spacetime lattice. We assume spacetime to be filled with a cubic lattice with lattice constant $\Delta z = c\Delta t = l_P$. Our approach[64] bears some resemblance to Feynman's[65] and is based on Dirac's observation[66] that the instantaneous velocity operators of the spin-$\frac{1}{2}$ particle (hereafter called by the generic name "the electron") have eigenvalues $\pm c$ and that they anticommute. We assume that the electron of mass m moves with the speed of light ($c = 1$ hereafter) from one lattice site to a neighboring (spatially left or right) site with time t always increasing on the "cubic" spacetime (z,t) lattice. The wavefunction has two components

$$\psi(z,t) = \begin{pmatrix} \psi_+(z,t) \\ \psi_-(z,t) \end{pmatrix}, \tag{38}$$

where ψ_+ denotes the component arriving from the event $(z - l_P, t - l_P)$ while ψ_- means arriving from $(z + l_P, t - l_P)$.

Next we assume that, at the lattice site (z,t), the arriving components are partially turned around by a unitary matrix:

$$\begin{pmatrix} \psi_+(z,t) \\ \psi_-(z,t) \end{pmatrix} = \mathcal{F} \begin{pmatrix} \psi_+(z - l_P, t - l_P) \\ \psi_-(z + l_P, t - l_P) \end{pmatrix}, \tag{39}$$

with the "flip operator" $\mathcal{F}$ defined by

$$\mathcal{F} \equiv e^{-iFml_P}. \tag{40}$$

Here F is a hermitian 2×2 matrix which we give the most obvious form

$$F = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{41}$$

with σ_1 being the first Pauli matrix. We will show that, in the limit $l_P \to 0$, the lowest nontrivial term of Eq. (39) yields the Dirac equation. We first write

$$\begin{pmatrix} \psi_+(z - l_P, t - l_P) \\ \psi_-(z + l_P, t - l_P) \end{pmatrix} = \mathcal{T}\psi(z,t), \tag{42}$$

with the "transfer" operator $\mathcal{T}$ given by

$$\mathcal{T} = e^{-l_P\left(\frac{\partial}{\partial t} + \sigma_3 \frac{\partial}{\partial z}\right)}, \tag{43}$$

where $\sigma_3 \equiv diag(1, -1)$ is the third Pauli matrix. Then the difference equation Eq. (39) takes the form

$$\psi(z,t) = \mathcal{F}\mathcal{T}\psi(z,t). \tag{44}$$

The difference equation becomes a differential equation if we limit ourselves to the zeroth order (given by the identity $\psi(z,t) = \psi(z,t)$) and the first order term in l_P. The first order equation is

$$i\frac{\partial}{\partial t}\psi(z,t) = m\sigma_1\psi(z,t) - i\sigma_3\frac{\partial}{\partial z}\psi(z,t), \tag{45}$$

which is the Dirac equation in $(1+1)$ dimensions!

In the scenario we have proposed, the electron travels between lattice sites with the speed of light. We[64] speculate that only at the lattice sites does the electron "feel" its mass and perhaps also the presence of all external fields. But if gravitational interactions also take place mainly at the lattice sites, does that mean spacetime vertices somehow play an important role in concentrating curvature? And if so, how is this description of geometry and topology related to the Regge calculus[67], for example? These problems deserve further invesigation.

Appendix B: Λ as an interplay between Planck- and Hubble-scale physics

Recent astrophysical observations indicate that the cosmological constant is probably small but nonzero; it is positive, giving rise to cosmic repulsion, and is the source of the dark energy, accounting for about 70% of the total cosmic (critical) energy density. In this appendix, we examine a scenario which suggests that the dark energy may come about as a result of the interplay between Planck-scale and Hubble-scale physics. The idea makes crucial use of the theory of unimodular gravity[68, 69], which, for the purpose of this Brief

Report, can be regarded as the ordinary theory of gravity except for the way the cosmological constant Λ arises in the theory.

We use the version of unimodular gravity given by the Henneaux and Teitelboim action[70]

$$S_{unimod} = -\frac{1}{16\pi G}\int[\sqrt{g}(R+2\Lambda) - 2\Lambda\partial_\mu\mathcal{T}^\mu](d^3x)dt. \tag{46}$$

One of its equations of motion is $\sqrt{g} = \partial_\mu\mathcal{T}^\mu$, the generalized unimodular condition, with g given in terms of the auxiliary field $\mathcal{T}^\mu$. Note that, in this theory, Λ/G plays the role of "momentum" conjugate to the "coordinate" $\int d^3x\mathcal{T}_0$ which can be identified, with the aid of the generalized unimodular condition, as the spacetime volume V. Hence Λ/G and V are conjugate to each other. It follows that their fluctuations obey a Heisenberg-type quantum uncertainty principle,

$$\delta V\delta\Lambda/G \sim 1. \tag{47}$$

Next we borrow an argument due to Sorkin[71], drawn from the causal-set theory, which stipulates that continous geometries in classical gravity should be replaced by "causal-sets", the discrete substratum of spacetime. In the framework of the causal-set theory, the fluctuation in the number of elements N making up the set is of the Poisson type, i.e., $\delta N \sim \sqrt{N}$. For a causal set, the spacetime volume V becomes $l_P^4 N$. It follows that

$$\delta V \sim l_P^4\delta N \sim l_P^4\sqrt{N} \sim l_P^2\sqrt{V} = G\sqrt{V}. \tag{48}$$

Putting Eqs. (47) and (48) together yields a minimum uncertainty in Λ given by $\delta\Lambda \sim V^{-1/2}$. By following an argument due to Baum[72] and Hawking[73], it has been plausibly argued[69] that, in the framework of unimodular gravity, Λ vanishes to the lowest order of approximation (in the "late" Universe) and that it is positive if it is not zero. So we conclude that Λ is positive and it fluctuates about zero with a magnitude[74] of $V^{-1/2} \sim R_H^{-2}$, where R_H is the Hubble radius of the Universe, contributing an energy density ρ given by:

$$\rho \sim +\frac{1}{l_P^2 R_H^2}, \tag{49}$$

which is of the order of the critical density as observed! The appearance of both the Planck length (the smallest length scale) and the Hubble radius (the largest observable scale) in ρ seems to suggest that the dark energy is due to an interplay between ultraviolet and infrared physics.

[1] See, e.g., J. Polchinski, *String Theory* (Cambridge University, 1998), and references therein.

[2] See, e.g., R. Gambini and J. Pullin, *Loops, knots, gauge theories and quantum gravity* (Cambridge University, 1996), and references therein.

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